Toribash Head Texture Vertical Pixel Stretch Factor

Anthony Cox

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Abstract

When you map a planar texture onto a sphere, the pixels are stretched vertically with more distortion towards the poles of the spheres. This document describes a formula to calculate the vertical stretching of the pixels.

This document was written as a memorandum for a friend and not as a definitive piece of mathematical work. Use of the content of this document is entirely at your own risk.

1 Introduction

The formulas and methods described in this document relate to head textures in the game Toribash. As such, they refer to planar textures of 128px x 128px mapped onto a sphere. These formulas can be used for other texture sizes, but changes will have to be made for the constant values in them.

1.1 Motivation

Through the use of these algorithms it is possible to more accurately map head textures for use in Toribash.

2 Formulas

As the pixel stretch factor is the same along an entire row of pixels on the planar texture, it is possible to model with a circle rather than a sphere for the sake of simplicity.

The x coordinate of a point on the circumference of a circle with the y coordinate corresponding with the row of pixels we are interested in on the planar texture first needs to be calculated. A simple application of pythagoras' theorem will yield the result since the radius of the circle is known. The y value used in Equation 1 is measured in texture coordinates (0 at the top) rather than from the origin of the circle.

$$\begin{aligned} h^2 &= x^2 + y_1^2 \\ x &= \sqrt{h^2 - y_1^2} \\ &= \sqrt{64^2 - (64 - y)^2} \end{aligned} \tag{1}$$

The angle of the gradient at this point on the sphere can be used for the pixel stretch factor and consists of calculating the derivative of the circle.

$$\begin{aligned} \frac{dy}{dx} &= \frac{a - x}{y - b} \\ &= \frac{-x}{y - 64} \end{aligned} \tag{2}$$

The derivative from Equation 2 is now made to correspond to the amount of vertical stretching applied to the pixels when mapping to the sphere simply by calculating the inverse.

$$\delta = \frac{1}{|\frac{dy}{dx}|} + 1 \tag{3}$$

δ is the pixel stretch factor at the given y coordinate. This value ranges from 1 at the equator to ∞ at the pole of the sphere. At 45° the stretch factor will be 2 and corresponds to a doubling in the height of the pixels when viewed from a point collinear with the poles of the sphere rather than from a point which lies on the equatorial plane.

The given formulas have a range of $0 \leq y < 64$. For $64 < y \leq 128$ apply the function $y = 128 - y_1$. When $y = 64$, $\delta \equiv 1$